International Journal of Research Instinct
(Peer-reviewed Journal)

ISSN ONLINE 2348-2095
Research Article

LEHMER-3 MEAN LABELING OF CYCLE RELATED GRAPHS

GOPI R [1] and NIRMALA C [2]
Department of Mathematics, Srimad Andavan Arts and Science College (Autonomous),
Tiruchirappalli – 620005.
[1]drrgmaths@gmail.com, [2]nimmikumarsan@gmail.com

ABSTRACT

A graph $G = (V, E)$ with p vertices and q edges is called a Lehmer-3 mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2,\ldots,q+1$ in such a way that when each edge $e = uv$ is labeled with $f^*(uv) = \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil (or) \left\lfloor \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rfloor$ then the edge labels are distinct. In this case f^* is called a Lehmer-3 mean labeling of G. In this paper we prove that Lehmer-3 mean labeling of cycle related graphs.AMS subject classification: 05C78

Keywords: Lehmer-3 mean labeling, Lehmer-3 mean graph, Corona.

1.INTRODUCTION

The graphs considered here are simple, finite and undirected graph. The graph $G = (V,E)$ has p vertices and q edges. For a detailed survey of graph labeling, we refer to Gallian [1]. For all other standard terminology and notations, we follow Harary [2]. The concept of Lehmer-3 mean labeling was introduced by S. Somasundaram, S.S. Sandhya and T.S. Pavithra [3].

2. Main Results

Definition 2.1: The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Theorem 2.2

The Graph $P_m \Theta C_n$ is a Lehmer-3 Mean Graph for any $m, n \geq 3$.

Proof:

Let $\{u_j, 1 \leq j \leq m, v_{ij}, 1 \leq i \leq n-1, 1 \leq j \leq m\}$ be the vertices and

$\{e_{j,} 1 \leq j \leq m-1, e_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$ be the edges which are denoted as in Figure 1.1

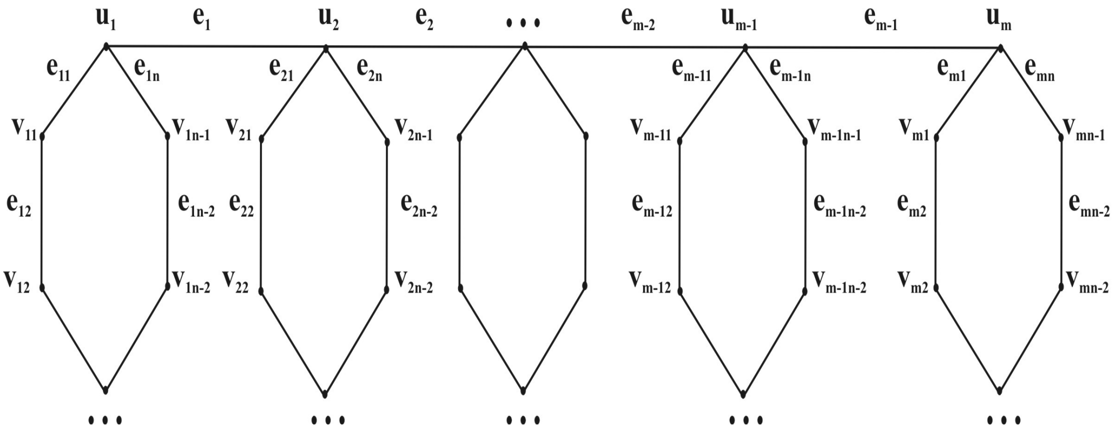

Figure 1.1: Ordinary labeling of $P_m \Theta C_n$

Case(i)n is odd

First we label the vertices as follows:

For $1 \leq j \leq m$

$$f(u_j) = (n+1)(j-1)+1$$

For $1 \leq j \leq m$

$$f(v_{ij}) = \begin{cases} 2i + (n+1)(j-1) & for \ 1 \leq i \leq \dfrac{n-1}{2} \\ (n+1)(j-1) + 2n - 2i + 1 & for \ \dfrac{n+1}{2} \leq i \leq n-1 \end{cases}$$

Then the induced edge labels are:

For $1 \leq j \leq m-1$

$$f^*(e_j) = (n+1)j$$

For $1 \leq j \leq m$

$$f^*(e_{ij}) = \begin{cases} (j-1)(n+1)+2i-1 & for\ 1 \le i \le \frac{n-1}{2} \\ (j-1)(n+1)+2(n-i+1) & for\ \frac{n+1}{2} \le i \le n \end{cases}$$

Case (ii)n is even

First we label the vertices as follows:

For $1 \le j \le m$

$$f(u_j) = (n+1)(j-1)+1$$

For $1 \le j \le m$

$$f(v_{ij}) = \begin{cases} (j-1)(n+1)+2i & for\ 1 \le i \le \frac{n}{2} \\ (j-1)(n+1)+2n-2i+1 & for\ \frac{n+2}{2} \le i \le n-1 \end{cases}$$

Then the induced edge Labels are:

For $1 \le j \le m-1$

$$f^*(e_j) = (n+1)j$$

For $1 \le j \le m$

$$f^*(e_{ij}) = \begin{cases} (j-1)(n+1)+2i-1 & for\ 1 \le i \le \frac{n}{2} \\ (j-1)(n+1)+2(n-i+1) & for\ \frac{n+2}{2} \le i \le n \end{cases}$$

Therefore, the edge labels are all distinct. Hence, the graph $P_m \Theta C_n$ is a Lehmer-3 mean graph.

Example:

Lehmer-3 mean labeling of $P_5 \Theta C_5$ is shown in Figure 1.2

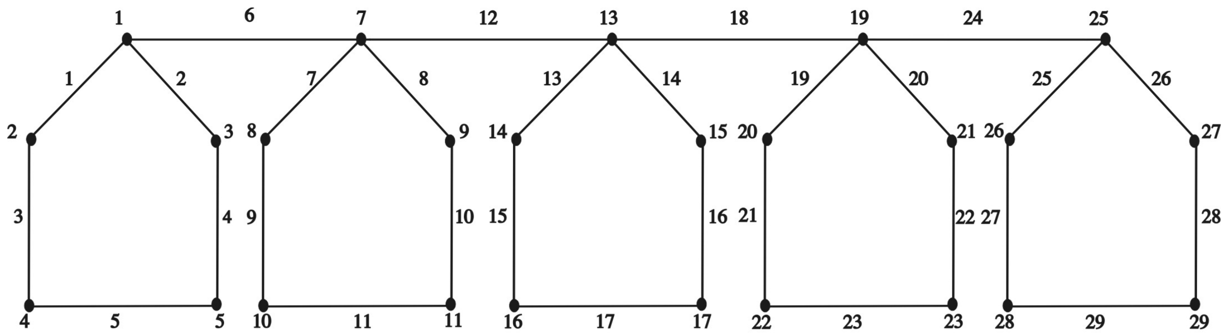

Figure 1.2: Lehmer-3 Mean labeling of $P_5 \Theta C_5$

Definition 2.3

The graph (P_m, C_n) is obtained from m copies of C_n and the path P_m:$u_1, u_2, u_3, \ldots, u_m$ by joining u_i with the vertex v of the i^{th}copy of C_n by means of an edge for $1 \le i \le m$.

Theorem 2.4

The Graph (P_m, C_n) is a Lehmer-3 Mean Graph for any $m, n \ge 3$.

Proof:

Let $\{u_j, w_j, 1 \le j \le m, v_{ij}, 1 \le i \le n-1, 1 \le j \le m\}$ be the vertices and $\{e_{j,} 1 \le j \le m-1, a_j, 1 \le j \le m, e_{ij}, 1 \le i \le n, 1 \le j \le m\}$ be the edges which are denoted as in Figure 1.3.

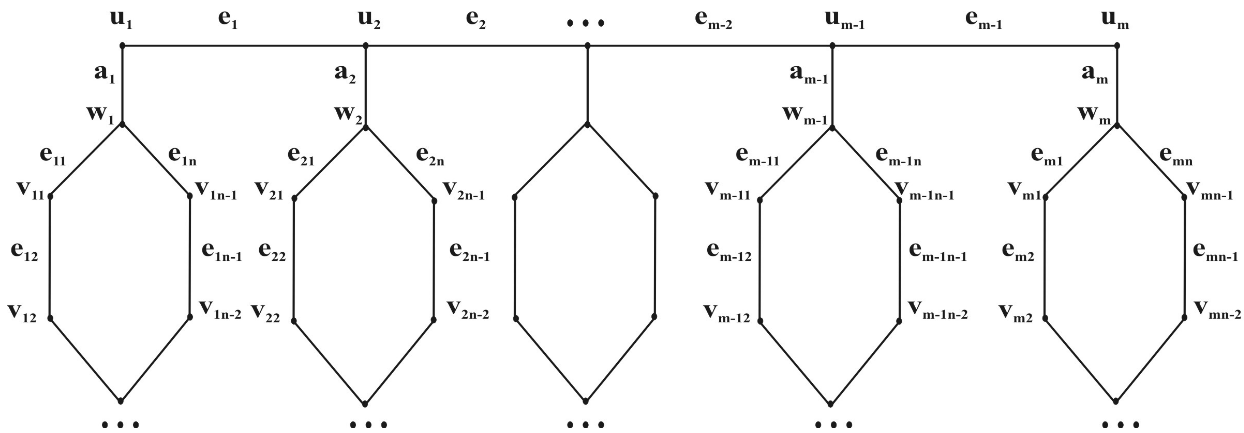

Figure 1.3: Ordinary labeling of (P_m, C_n)

Case (i)n is odd

First we label the vertices as follows:

For $1 \le j \le m$

$$f(u_j) = (j-1)(n+2)+1$$

$$f(w_j) = (j-1)(n+2)+2$$

For $1 \le j \le m$

$$f(v_{ij}) = \begin{cases} (j-1)(n+2)+2i+1 & for\ 1 \le i \le \dfrac{n-1}{2} \\ (j-1)(n+2)+2(n-i+1) & for\ \dfrac{n+1}{2} \le i \le n-1 \end{cases}$$

Then the induced edge labels are:

For $1 \le j \le m-1$

$$f^*(e_j) = (n+2)j$$

For $1 \le j \le m$

$$f^*(a_j) = (j-1)(n+2)+1$$

For $1 \le j \le m$

$$f^*(e_{ij}) = \begin{cases} (j-1)(n+2)+2i & for\ 1 \le i \le \dfrac{n+1}{2} \\ (j-1)(n+2)+(-2i+2n+3) & for\ \dfrac{n+1}{2} \le i \le n \end{cases}$$

Case (ii) n is even

First we label the vertices as follows:

For $1 \le j \le m$

$$f(u_j) = (j-1)(n+2)+1$$

$$f(w_j) = (j-1)(n+2)+2$$

For $1 \le j \le m$

$$f(v_{ij}) = \begin{cases} (j-1)(n+2)+2i+1 & for\ 1 \le i \le \dfrac{n}{2} \\ (j-1)(n+2)+2(n-i+1) & for\ \dfrac{n+2}{2} \le i \le n-1 \end{cases}$$

Then the induced edge labels are:

For $1 \le j \le m-1$ $f^*(e_j) = (n+2)j$

For $1 \le j \le m$ $f^*(a_j) = (j-1)(n+2)+1$

For $1 \le j \le m$

$$f^*(e_{ij}) = \begin{cases} (j-1)(n+2)+2i & for\ 1 \le i \le \dfrac{n}{2} \\ (j-1)(n+2)+(-2i+2n+3) & for\ \dfrac{n+2}{2} \le i \le n \end{cases}$$

Therefore, the edge labels are all distinct. Hence the graph (P_m, C_n) is a Lehmer-3 mean graph

Example:

Lehmer-3 mean labeling of (P_5, C_4) is shown in Figure 1.4

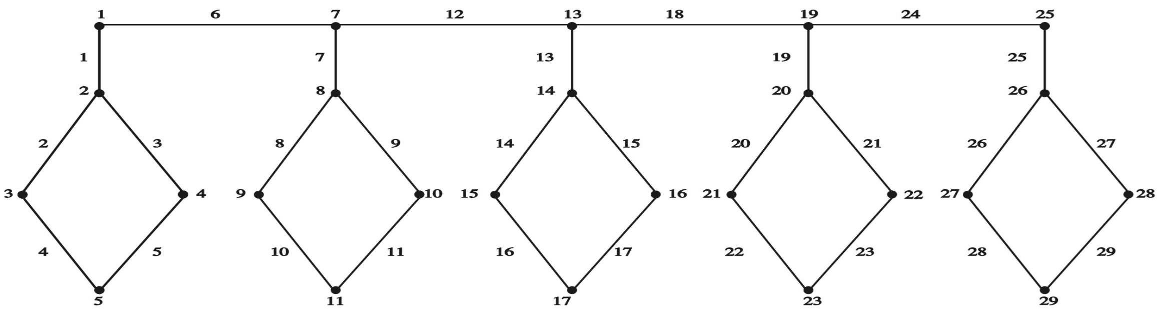

Figure 1.4: Lehmer-3 mean labeling of (P_5, C_4)

REFERENCES

[1] Gallian. J.A.A dynamic survey of graph labeling, The Electronic Journal of combinations, 18#DS6 (2016).

[2] Harary.F, Graph Theory, Narosa Publishing House Reading, New Delhi (1988).

[3] Somasundaram.S, Sandhya.S.S and Pavithra T.S, Lehmer-3 Mean Labeling of sDisconnected graphs, Asian Pacific Journal Of Research, Vol.1 XL, June 2016(198-204).