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Research Article

THE *M–N*-HOMOMORPHISM, *M–N*-ANTI HOMOMORPHISM OVER *M–N*-SOFT FUZZY SUBGROUPS

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Abstract

In this paper soft fuzzy set and soft fuzzy subgroup and the concept of *M–N*- fuzzy subgroups and some elementary properties are discussed. We also try to use soft fuzzy subgroup in M-N –fuzzy subgroup. We use soft fuzzy subgroup as a basic tool to find the *M–N*-soft fuzzy subgroup, *M–N*-normal soft fuzzy subgroup We introduce *M–N*-homomorphism and *M–N*-anti homomorphism over *M–N*-soft-fuzzy subgroup, also extend some results on this subject.

Key Words

Fuzzy subset, M–N-fuzzy subgroup, M–N- normal fuzzy subgroup, M–N-homomorphism, M–N-anti homomorphism, soft fuzzy subset, M–N-soft fuzzy subgroup, M–N-normal soft fuzzy subgroup.

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Introduction

L.A.zedeh [1] introduce the concepts of fuzzy sets and fuzzy operations. Later Solairaju and Nagarajan [2] introduced the notation of Q fuzzy groups. Also A.Sheik & K.Jeyaraman [3] define the anti homomorphism in groups and normal fuzzy subgroups. Then M.O.Massa'deh [4] discuss the M-N-fuzzy subgroups.

In this paper we have to develop all M-N- fuzzy subgroups and M-N- Homomorphism & M-N- Anti homomorphism over M-N- fuzzy subgroups and also discussed the M-N- soft fuzzy subgroups.

Preliminaries

Definition 2.1: Fuzzy set

Let X be a non-empty set. A fuzzy set μ on X is a mapping $\mu: X \to [0, 1]$ and is denoted by $\mu = \{(x, \mu(x))/x \in X\}$.

Definition 2.2: Level fuzzy set

Let μ be a fuzzy subset of a set X. For t $\in$ [0, 1], $\mu_l=\{(x \in X / \mu(x) \geq t\}$ is called level fuzzy subset of μ.

Definition 2.3: Image and Pre-image of a fuzzy set

Let X and Y be two sets. Let f : X$\rightarrow$Y be a function. If μ is a fuzzy set on X, then the image of μ under f is a fuzzy set on Y and is defined by $\{f(\mu)\}(y) = \sup\limits_{x \in f^{-1}(y)} \mu(x),\ \forall\ y \in Y$. Let λ be a fuzzy set on Y. The pre-image of S is a fuzzy set on X and is defined by $\{f^{-1}(\lambda)\}(x) = \lambda(f(x))$.

Definition 2.4: Soft set

Let U be an initial universe, P(U) be the power set of U, E be the set of all parameters and A $\subseteq$ E. A soft set (f_A, E) on the universe U is defined by the set of ordered pair, $(f_A, E) = \{(e, f_A(e));\ e \in E, f_A(e) \in P(U)\}$ where f_A:$E \rightarrow P(U)$ such that, $f_A(e) = \phi$ if $e \notin A$.

Example:

Let U= $\{s_1, s_2, s_3, s_4\}$ be a set of four shirts and E = {*white*(e_1), *red*(e_2), *blue*(e_3)} be a set of parameters. If A= $\{e_1, e_2\}$ $\subseteq$ E. Let $f_A(e_1)$= $\{s_1, s_2, s_3, s_4\}$, $f_A(e_2)$= $\{s_1, s_2, s_3\}$ and $f_A(e_3)$= ϕ since, $e_3 \notin A$. Then we write the soft set over U as follows. $(f_A, E) = \{(e_1, \{s_1, s_2, s_3, s_4\}), (e_2, \{s_1, s_2, s_3\})\}$.

We may represent the soft set in the following form,

U	e_1	e_2	e_4
s_1	1	1	0
s_2	1	1	0
s_3	1	1	0
s_4	1	0	0

Definition 2.5:

For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B) and we write $(F, \mathrm{A}) \subseteq (G, B)$.

If, (i) $A \subseteq B$

(ii) For each $a \in A$, $F(a) \subseteq G(a)$

Definition 2.6:

Two soft sets (F, A) and (G, B) over a common universe U are said to be equal if $(F, \mathrm{A}) \subseteq (G, B)$ and $(G, \mathrm{B}) \subseteq (F, \mathrm{A})$.

Definition 2.7:

Union of two soft sets (F,A) and (G,B) over a common universe X is the soft set (H,C), where $C = A \cup B$ and

$$H(c) = \begin{cases} F(c) & if \quad c \in A - B \\ G(c) & if \quad c \in B - A \qquad \forall\, c \in C \\ F(c) \cup G(c) & if \quad c \in A \cap B \end{cases}$$

We write,

$$(F,A) \cup (G,B) = (H,C).$$

Definition 2.8:

Intersection of two soft sets (F,A) and (G,B) over a common universe X is the soft set (H,C) where $C = A \cap B$ and $H(c) = F(c) \cap G(c) \quad \forall\, c \in C$

We write,

$$(F,A) \cap (G,B) = (H,C)$$

i.e. $F : A \to P(\mathrm{X})$

$G : B \to P(X)$

$H : C \to P(X)$

$H : A \cap B \to P(X)$

Definition 2.9: (AND)

If (F,A) and (G,B) are two soft sets over a common universe U, then "(F,A) AND (G,B)" denoted by $(F,\mathrm{A}) \tilde{\wedge} (G,B)$ is defined by $(F,A) \tilde{\wedge} (G,B) = (H, A \times B)$, where $H(x,y) = F(x) \cap G(y) \ \forall (x,y) \in A \times B$

Definition 2.10: (OR)

If (F,A) and (G,B) are two soft sets over a common universe U, then "(F,A) OR (G,B)" denoted by $(F,\mathrm{A}) \tilde{\vee} (G,B)$ is defined by $(F,A) \tilde{\vee} (G,B) = (H, A \times B)$, where $H(x,y) = F(x) \cup G(y) \ \forall (x,y) \in A \times B$

Definition 2.11: AND Soft set

Let $(F_i, A_i)_{i \in I}$ be a non-empty family of soft sets over a common universe U. The AND-soft set $\tilde{\wedge}_{i \in I} (F_i, A_i)$ of these soft sets is defined to be the soft set (H,B) such that $B = \prod_{i \in I} A_i$ and

$H(x) = \bigcap_{i \in I} F_i(x_i)$ for all $x = (x_i)_{i \in I} \in B$.

Definition 2.12: OR Soft set

Let $(F_i, A_i)_{i \in I}$ be a non-empty family of soft sets over a common universe set U. The OR-soft set $\tilde{\vee}_{i \in I} (F_i, A_i)$ of these soft sets is defined to be the soft set (H, C) such that $C = \prod_{i \in I} A_i$ and $H(x) = \bigcup_{i \in I} F_i(x_i)$ for all $x = (x_i)_{i \in I} \in C$.

Note: If $A_i = A$ and $F_i = F \ \forall\ i \in I$, then $\tilde{\wedge}_{i \in I} (F_i, A_i) = \tilde{\wedge}_{i \in I} (F, A)$.

In this case, $\prod_{i \in I} A_i = \prod_{i \in I} A$.

Definition 2.13: Fuzzy group

Let G be a group. A fuzzy subset μ of G is said to be a fuzzy subgroup of G if

(i) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x^{-1}) \geq \mu(x) \ \forall\ x, y \in G$.

Definition 2.14: Fuzzy level subgroup

Let μ be a fuzzy subgroup of a group G. The subgroup μ_t of G, for $t \in [0,1]$ such that $\mu(e) \geq t$ is called a level subgroup of μ.

Definition 2.15: Normal fuzzy subgroup

A fuzzy subgroup μ of a group G is called normal fuzzy subgroup. If $\mu(x^{-1}yx) \geq \mu(y) \ \forall\ x, y \in G$.

Definition 2.16: Soft group

Let X be a group and (F, A) be a soft set over X. Then (F, A) is said to be a soft group over X iff $F(a) < X \ \forall\ a \in X$.

Definition 2.17: Fuzzy soft group

Let X be a group and (μ, A) be a fuzzy soft set over X. Then (μ, A) is said to be a fuzzy soft group over X iff for each $a \in A$ & $x, y \in X$,

(i) $\mu_a(x, y) \geq \min\{\mu_a(x), \mu_a(y)\}$

(ii) $\mu_a(x^{-1}) \geq \mu_a(x)$

That is, for each $a \in A$, μ_a is a fuzzy subgroup in Rosenfield's sense.

3. $M-N-$Fuzzy and Normal Fuzzy Subgroups

Definition 3.1: $M-N-$Fuzzy subgroups

Let G be an $M-N-$group and μ be a fuzzy subgroup of G. If,

1. $\mu(mx) \geq \mu(x)$
2. $\mu(xn) \geq \mu(x)$

Hold for any $x \in G$, $m \in M$ & $n \in N$, then μ is said to be an $M-N-$fuzzy subgroup of G.

Definition 3.2:

Let G be an $M-N-$group. μ is said to be an $M-N-$normal fuzzy subgroup of G if μ is not only an $M-N-$fuzzy subgroup of G, but also normal fuzzy subgroup of G.

Proposition 3.3:

Let G be an $M-N-$group. μ,λ both be $M-N-$fuzzy subgroups of G, then the intersection of μ,λ is an $M-N-$fuzzy subgroup of G.

.Corollary 3.4:

The intersection of two $M-N-$normal fuzzy subgroups μ,λ is an $M-N-$normal fuzzy subgroup of G.

Corollary 3.5:

If μ is an $M-N-$fuzzy subgroup of an $M-N-$group G, then the following statement hold for all $x,y \in G$, $m \in M$ and $n \in N$.

1. $\mu\left(\left(m(xy)n\right)\right) \geq \min\left\{\mu(x),\mu(y)\right\}$
2. $\mu\left(\left(mx^{-1}\right)n\right) \geq \mu(x)$

Theorem 3.6:

Let G be an $M-N-$group, λ be a fuzzy set of G. Then λ is $M-N-$fuzzy subgroup of G iff for any $t \in [0,1]$, λ_t is an $M-N-$subgroup of G, when $\lambda_t \neq \phi$.

Corollary 3.7:

Let μ be a fuzzy set of the $M-N-$group of G, then μ is an $M-N-$normal fuzzy subgroup iff μ_t is an $M-N-$normal subgroup of G for any $t \in [0,1]$, $\mu_t \neq \phi$.

Definition 3.8: Let μ be an $M-N-$fuzzy subgroup of an $M-N-$group G and let $H=\{x\in G, m\in M, n\in N; \mu(mxn)=\mu(e)\}$. Then μ is an $M-N-$fuzzy abelian subgroup of G.

4. $M-N-$Homomorphism and $M-N-$Anti-Homomorphism

Definition 4.1: Let G_1 and G_2 both be $M-N-$groups and ψ be a homomorphism from G_1 onto G_2. If $\psi(mx)=m\psi(x)$ and $\psi(xn)=\psi(x)n$ for all $x\in G_1, m\in M$ and $n\in N$. Then ψ is called an $M-N-$homomorphism.

Proposition 4.2: Let G_1 and G_2 both be $M-N-$groups and ψ be a homomorphism from G_1 onto G_2. If μ is an $M-N-$fuzzy subgroup of G_2, then $\psi^{-1}(\mu)$ is an $M-N-$fuzzy subgroup of G_1.

Corollary 4.3: Let G_1 and G_2 both be $M-N-$groups and ψ be a homomorphism from G_1 onto G_2. If μ is an $M-N-$normal fuzzy subgroup of G_2, then $\psi^{-1}(\mu)$ is an $M-N-$normal fuzzy subgroup of G_1.

Proposition 4.4: Let G_1 and G_2 both be $M-N-$groups and ψ be a homomorphism from G_1 onto G_2. If μ is an $M-N-$fuzzy subgroup of G_1, then $\psi(\mu)$ is an $M-N-$fuzzy subgroup of G_2.

Corollary 4.5: Let G_1 and G_2 both be $M-N-$groups and ψ be a homomorphism from G_1 onto G_2. If μ is an $M-N-$normal fuzzy subgroup of G_1, then $\psi(\mu)$ is an $M-N-$normal fuzzy subgroup of G_2.

Definition 4.6: Let G_1 and G_2 both be $M-N-$groups, then the function ψ from G_1 onto G_2 is said to be $M-N-$anti homomorphism. If $\psi\left(m(xy)\right)=m\psi(y)\psi(x)$ and $\psi\left((xy)n\right)=\psi(y)\psi(x)n$ for all $x\in G_1$, $m\in M$ and $n\in N$.

Definition 4.7: Let μ be an $M-N-$fuzzy characteristic subgroup of $M-N-$group G if $\mu\left(\psi\left(m(xy)n\right)\right)=\mu\left(m(xy)n\right)$.

Definition 4.8: Soft fuzzy $M-N-$ subgroup (New)

Let G be an $M-N-$group and (μ, A) be a soft fuzzy subgroup of G. If ,

1. $\mu_a\{m(xy)n\}\geq \min\{\mu_a(x), \mu_a(y)\}$
2. $\mu_a\{(mx^{-1})n\}\geq \mu_a(x)$

Hold for any $x, y \in G$, $m \in M$ and $n \in N$, then (μ, A) is said to be an $M-N-$soft fuzzy subgroup of G. Here $\mu_a : A \to P(G)$.

5. $M-N-$Soft Fuzzy and Normal Soft Fuzzy Subgroups

Definition 5.1:

Let G be an $M-N-$group and (μ, A) be a soft fuzzy subgroup of G. If

1. $\mu_a(mx) \geq \mu_a(x)$
2. $\mu_a(xn) \geq \mu_a(x)$

Hold for any $x \in G$, $m \in M$ & $n \in N$, then (μ, A) is said to be an $M-N-$soft fuzzy subgroup of G.

Definition 5.2: Let G be an $M-N-$group. (μ, A) is said to be an $M-N-$normal soft fuzzy subgroup of G if (μ, A) is not only an $M-N-$soft fuzzy subgroup of G, but also normal soft fuzzy subgroup of G.

Proposition 5.3: Let G be an $M-N-$group. $(\mu, A),(\lambda, B)$ both be $M-N-$soft fuzzy subgroups of G, then the intersection of $(\mu, A),(\lambda, B)$ is an $M-N-$soft fuzzy subgroup of G.

Corollary 5.4: The intersection of to $M-N-$normal soft fuzzy subgroups $(\mu, A),(\lambda, B)$ is an $M-N-$normal soft fuzzy subgroup of G.

Corollary 5.5: If (μ, A) is an $M-N-$soft fuzzy subgroup of an $M-N-$group G, then the following statement hold for all $x, y \in G$, $m \in M$ and $n \in N$.

1. $\mu_a\left(\left(m(xy\right)n\right) \geq \min\{\mu_a(x), \mu_a(y)\}$
2. $\mu_a\left(\left(mx^{-1}\right)n\right) \geq \mu_a(x)$

Theorem 5.6:

Let G be an $M-N-$group, (λ, A) be a soft fuzzy set of G. Then (λ, A) is $M-N-$soft fuzzy subgroup of G iff for any $t \in [0,1]$, (λ_t, A) is an $M-N-$soft subgroup of G, when $(\lambda_t, A) \neq \phi$

Corollary 5.7:

Let (μ, A) be a soft fuzzy set of the $M-N-$group of G, then (μ, A) is an $M-N-$normal soft fuzzy subgroup iff (μ_t, A) is an $M-N-$normal soft subgroup of G for any $t \in [0,1]$, $(\mu_t, A) \neq \phi$.

Definition 5.8:

Let (μ, A) be an $M-N-$soft fuzzy subgroup of an $M-N-$group G and let $H = \{x \in G, m \in M, n \in N; \mu_a(mxn) = \mu_a(e)\}$. Then (μ, A) is an $M-N-$soft fuzzy abelian subgroup of G.

6. $M-N-$ Homomorphism and $M-N-$ Anti- Homomorphism of $M-N-$ soft fuzzy subgroups

Proposition 6.1:

Let G_1 and G_2 both be $M-N-$groups and f be a homomorphism from G_1 onto G_2. If (μ, A) is an $M-N-$soft fuzzy subgroup of G_2, then $f^{-1}(\mu)$ is an $M-N-$soft fuzzy subgroup of G_1.

Corollary 6.2:

Let G_1 and G_2 both be $M-N-$groups and f be a homomorphism from G_1 onto G_2. If (μ, A) is an $M-N-$normal soft fuzzy subgroup of G_2, then $f^{-1}(\mu)$ is an $M-N-$normal soft fuzzy subgroup of G_1.

Proposition 6.3:

Let G_1 and G_2 both be $M-N-$groups and f be a homomorphism from G_1 onto G_2. If (μ, A) is an $M-N-$soft fuzzy subgroup of G_1, then $f(\mu)$ is an $M-N-$soft fuzzy subgroup of G_2.

Corollary 6.4:

Let G_1 and G_2 both be $M-N-$groups and f be a homomorphism from G_1 onto G_2. If (μ, A) is an $M-N-$normal soft fuzzy subgroup of G_1, then $f(\mu)$ is an $M-N-$normal soft fuzzy subgroup of G_2.

Definition 6.5:

Let G_1 and G_2 be $M-N-$groups. The function f from G_1 onto G_2 is said to be $M-N-$ anti homomorphism, if $f(m(xy)) = mf(y)f(x)$ and $f((xy)n) = f(y)f(x)n$ for all $x \in G_1$, $m \in M$

and $n \in N$.

Definition 6.6:

(μ, A) be an $M-N-$soft fuzzy characteristic subgroup of $M-N-$group G if $\mu_a\left(f\left(m(xy\right)n\right) = \mu_a\left(m(xy)n\right)$.

7. $M-N-$Homomorphism and $M-N-$Anti homomorphism over $M-N-$Soft Fuzzy Subgroups

Theorem 7.1:

Let $f: G_1 \to G_2$ be an $M-N-$anti homomorphism. If (μ, A) is an $M-N-$soft fuzzy subgroup of G_2, then $f^{-1}(\mu)$ is an $M-N-$soft fuzzy subgroup of G_1.

Proof: Let $x, y \in G_1$, $m \in M$, $n \in N$

$$f^{-1}(\mu_a)\left(m(xy)\right) = \mu_a\left(f(m(xy)\right) = \mu_a\left(m(f(y)(x)\right)$$

$$\geq \min\left\{\mu_a(m(f(y)), \mu_a(f(x))\right\}$$

$$\geq \min\left\{mf^{-1}(\mu_a(y)), f^{-1}(\mu_a(x))\right\}$$

And

$$f^{-1}(\mu_a)\left((xy)n\right) = \mu_a\left(f((xy)n\right) = \mu_a\left((f(y)f(x)n\right)$$

$$\geq \min\left\{\mu_a((f(y)), \mu_a(f(x)n)\right\}$$

$$\geq \min\left\{f^{-1}(\mu_a(y)), f^{-1}(\mu_a(x)n)\right\}$$

Also $f^{-1}\left(\mu_a(mx^{-1})n\right) = \mu_a\left(mf((x^{-1})n\right) = \mu_a\left(mf((x)n\right) = \mu_a\left(f(x)\right) = f^{-1}\left(\mu_a(x)\right)$.

Therefore $f^{-1}(\mu_a)$ is $M-N-$soft fuzzy subgroup of G_1.

Corollary 7.2:

Let $f: G_1 \to G_2$ be an $M-N-$anti homomorphism. If (μ, A) is an $M-N-$normal soft fuzzy subgroup of G_2, then $f^{-1}(\mu, A)$ is an $M-N-$normal soft fuzzy subgroup of G_1.

Proof: Let $x, y \in G_1$, $m \in M$, $n \in N$ by theorem 7.1 $f^{-1}(\mu_a)$ is an $M-N-$soft fuzzy subgroup of G_1. $f^{-1}(\mu_a)\left((xy)\right) = \mu_a\left(f(xy)\right) = \mu_a\left((f(y)f(x)\right) = \mu_a\left(f(yx)\right) = f^{-1}(\mu_a)\left((yx)\right)$.

Which is implies that $f^{-1}(\mu_a)$ is an $M-N-$normal soft fuzzy subgroup of G_1.

Theorem 7.3:

An $M-N-$soft fuzzy characteristic subgroup on $M-N-$soft fuzzy subgroup is an $M-N-$normal soft fuzzy subgroup.

Proof:

Let f be an $M-N-$anti homomorphism of G which is implies that $f(xy)=f(y)f(x)$.

$$f(m(xy))=mf(y)f(x)$$

$$f((xy)n)=f(y)f(x)n$$

Since $\mu_a\left(m(xy)n\right)=\mu_a\left(f(m(xy)n\right)$ and

$$\mu_a\left(m(xy)n\right)=\mu_a\left(mf(y)f(x)n\right)=\mu_a\left(f(m(yx)n\right).$$

Hence μ_a is an $M-N-$normal soft fuzzy subgroup of G.

Theorem 7.4:

An $M-N-$anti homomorphism pre image of an $M-N-$soft fuzzy abelian subgroup is an $M-N-$soft fuzzy abelian subgroup.

Proof:

Let (λ, A) be an $M-N-$soft subgroup of G_1, we need to prove (λ, A) is an $M-N-$soft fuzzy abelian subgroup of G_1 suppose that (μ, A) is an $M-N-$soft fuzzy abelian subgroup of G_2. Then $H_2=\left\{y\in G_2, m\in M, n\in N; \mu_a\left(myn\right)=\mu_a(e_2)\right\}$ is an $M-N-$soft fuzzy abelian subgroup of G_2, where e_2 is the identity of G_2. Consider the set $H_1=\left\{x\in G_1, m\in M, n\in N; \lambda_a\left(mxn\right)=\lambda_a(e_1)\right\}$ where e_1 is the identity of G_1. Let $m(xy)n\in H_1\subseteq G_1$, then $\lambda_a(mxyn)=\lambda_a(e_1)$

$$\mu_a\left(f(m(xy)n\right)=\mu_a\left(f(e_1)\right)$$

$$\mu_a\left(f(m(xy)n\right)=\mu_a(e_2)$$

$$\mu_a\left(m(f(y)f(x)n\right)=\mu_a(e_2)$$

$m(f(y)f(x)n)\in H_2$ and H_2 is abelian, thus

$$\mu_a\left(m(f(y)f(x))n\right)=\mu_a\left(m(f(x)f(y))n\right)=\mu_a\left(mf(xy)n\right)=\mu_a\left(mf(yx)n\right)$$

$$\lambda_a\left(m(xy)n\right)=\lambda_a\left(m(yx)n\right)$$

$$\lambda_a(e_1)=\lambda_a\left(m(yx)n\right)$$

Therefore H_1 is an $M-N-$abelian subgroup and (λ, A) is an $M-N-$soft fuzzy abelian subgroup of G_1.

Theorem 7.5:

An $M-N-$anti homomorphism image of an $M-N-$soft fuzzy abelian subgroup is an $M-N-$soft fuzzy abelian subgroup.

Proof:

Let (λ, A) be an $M-N-$soft subgroup of G_2, we need to prove (λ, A) is an $M-N-$soft fuzzy abelian subgroup of G_2, suppose that f is an $M-N-$anti homomorphism from G_1 into G_2 since is an $M-N-$soft fuzzy abelian subgroup of G_1. Then $H_1=\left\{x\in G_1, m\in M, n\in N; \lambda_a\left(mxn\right)=\lambda_a(e_1)\right\}$ is an $M-N-$abelian subgroup of G_1 where e_1 is the identity of G_1. Let μ_a be an $M-N-$soft fuzzy abelian subgroup of G_2 and $H_2=\left\{y\in G_2, m\in M, n\in N; \mu_a\left(myn\right)=\mu_a(e_2)\right\}$ is an $M-N-$soft fuzzy abelian subgroup of G_2, e_2 is the identity of G_2. If $m(xy)n\in H_2\subseteq G_2, \mu_a\left(mxyn\right)=\mu_a(e_2)$

$$\sup_{z\in f^{-1}(m(xy)n)} \lambda_a(z)=\sup_{z\in f^{-1}(e_2)} \lambda_a(z)$$

$$\lambda_a\left(m(xy)n\right)=\lambda_a(e_1)$$

then $m(xy)n\in H_1$ and H_1 is an $M-N-$abelian subgroup, thus $\lambda_a\left(m(xy)n\right)=\lambda_a\left(m(yx)n\right)$

$$\sup_{z\in f^{-1}(m(xy)n)} \lambda_a(z)=\sup_{z\in f^{-1}(m(yx)n)} \lambda_a(z)$$

$$\mu_a\left(m(xy)n\right)=\lambda_a\left(m(yx)n\right)$$

$$\mu_a(e_2)=\mu_a\left(m(yx)n\right)$$

Therefore H_2 is an $M-N-$abelian subgroup of G_2 and μ_a is an $M-N-$soft fuzzy abelian subgroup of G_2.

Theorem 7.6:

Let f be an $M-N-$homomorphism from an $M-N-$group G_1 onto an $M-N-$group G_2. If (λ, A) is an $M-N-$soft fuzzy subgroup of G_1 and (λ, A) is an $f-$invariant, then $f(\lambda, A)$ is an $M-N-$soft fuzzy subgroup of G_2.

Proof:

Let $t\in I\,mf(\lambda_a)$, then for some $y\in G_2$.

$$\left(f(\lambda_a)\right)\left(y\right)=\sup_{x\in f^{-1}}\lambda_a(x)=t\,;$$

Where $t\le\lambda_a(e)$.

We know $\left(\lambda_a\right)_t$ is an $M-N-$soft subgroup of G_1 , if $t=1$ then $\left(f(\lambda_a)_t\right)=G_2$. If $0<t<1$, then $\left(f(\lambda_a)_t\right)=f\left(\left(\lambda_a\right)_t\right)$, since $z\in\left(f(\lambda_a)_t\right)\Leftrightarrow f(\lambda_a)(z)\ge t\Leftrightarrow\sup_{x\in f^{-1}(z)}\lambda_a(x)\ge t$

Iff there exist $x\in G_1$ such that $f(x)=z$ and $\lambda_a(x)\ge t$ iff $z\in\left(f\left(\lambda_a\right)_t\right)$.

Hence $\left(f(\lambda_a)_t\right)=f\left(\left(\lambda_a\right)_t\right)$ and is an $M-N-$homomorphism, $\left(f(\lambda_a)\right)$ is an $M-N-$soft subgroup of G_2 therefore $\left(f(\lambda_a)_t\right)$ is an $M-N-$soft subgroup of G_2 and $f(\lambda_a)$ is an $M-N-$ soft fuzzy subgroup of G_2 .

Theorem 7.7:

Let f be an $M-N-$anti homomorphism from an $M-N-$group G_1 onto an $M-N-$ group G_2 . If $\left(\lambda,A\right)$ is an $M-N-$soft fuzzy subgroup of G_1 and $\left(\lambda,A\right)$ is an $f-$invariant, then $f(\lambda_a)$ is an $M-N-$soft fuzzy subgroup of G_2 .

Proof:

Let $t\in I\,mf(\lambda_a)$, then for some $y\in G_2$.

$$\left(f(\lambda_a)\right)\left(y\right)=\sup_{x\in f^{-1}}\lambda_a(x)=t$$

Where $t\le\lambda_a(e)$.

We know $\left(\lambda_a\right)_t$ is an $M-N-$soft subgroup of G_1 , if $t=1$ then $\left(f(\lambda_a)_t\right)=G_2$. If $0<t<1$, then $\left(f(\lambda_a)_t\right)=f\left((\lambda_a)_t\right)$, since $z\in\left(f(\lambda_a)_t\right)\Leftrightarrow f(\lambda_a)(z)\ge t\Leftrightarrow\sup_{x\in f^{-1}(z)}\lambda_a(x)\ge t$

Iff there exist $x\in G_1$ such that $f(x)=z$ and $\lambda_a(x)\ge t$ iff $z\in\left(f(\lambda_a)\right)$.

Hence $\left(f(\lambda_a)_t\right)=f(\lambda_a)_t$ and is an $M-N-$anti homomorphism, $\left(f(\lambda_a)\right)$ is an $M-N-$soft subgroup of G_2 therefore $\left(f(\lambda_a)_t\right)$ is an $M-N-$soft subgroup of G_2 and $f(\lambda_a)$ is an $M-N-$ soft fuzzy subgroup of G_2 .

Conclusion

In this paper we have discussed M-N- Homomorphism soft fuzzy set group and M-N- Anti homomorphism soft fuzzy subgroup. Interestingly it has been observed that fuzzy concept adds an

M-N- fuzzy subgroups from defined fuzzy normal subgroups. The purpose of this paper we introduce the theory of M-N- soft fuzzy subgroups.

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