



ON STRONGLY g^* CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

This article introduces the concept of strongly generalized star continuous functions and strongly generalized irresolute functions and also studies their properties. Using these new types of continuous functions, several characterizations and properties have been obtained

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1.INTRODUCTION

In 1970 Levine[1] introduced and investigated the concept of generalized closed sets in topological spaces . After this work many of the authors investigated various forms of stronger and weaker forms of closed sets . In later [2] introduced strongly b^* closed sets in topological spaces .K.Balachandran [7] introduced generalized continuous maps in topological spaces .Homeomorphism plays a very important role in topology . By definition , a continuous function between two topological spaces X and Y is , if $f^{-1}(V)$ is closed in X , for every closed set V in Y .In 1995 , Maki , Devi and Balachandran [3] introduced the concepts of semi-generalized homeomorphisms and generalized semi homeomorphisms and studied some semi topological properties . Devi and Balachandran [4] introduced a

generalization of α -homeomorphism in 2001 . In this paper I first introduced a new type of continuous function called Sg^* -continuous function and a new type of irresolute function called Sg^* -irresolute function which are stronger than continuous functions and irresolute functions and also study their properties

2. PRELIMINARIES

Throughout this paper (X, τ) represent a topological space on which no separation axioms are assumed unless otherwise mentioned . For a subset A of a space X , $cl(A)$ denotes the closure of A and $int(A)$ denotes the interior of A respectively . A^c denotes the complement of A in X

Definition:2.1 A subset (X, τ) is said to be . A^c denotes the complement of A in X

- (1) Semi-pre closed (β -closed)[6] set if $int(cl(int(A))) \subseteq A$
- (2) θ -closed [12] set , if $A=cl \theta(A)$, where $cl \theta(A)=\{x \in X : cl(U) \cap A \neq \varnothing, U \in \tau, x \in U\}$
- (3) g -closed[6] set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X
- (4) δ -closed[12] set if $A=cl (A) \delta$, where $cl \delta (A)=\{x \in X : int(cl(U)) \cap A \neq \varnothing, U \in \tau, x \in U\}$
- (5) α -closed[4] set if $cl(int(cl(A))) \subseteq A$
- (6) wg -closed[5] set if $cl(int(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X
- (7) g^* -closed[6] set if if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g -open in X
- (8) b -closed[9] set if $(cl(int(A)) \cup int(cl(A))) \supseteq A$
- (9) b^{**} -closed[2] set if $A \subseteq (int(cl(int(A))) \cup (cl(int(cl(A))))$
- 10) g^* -closed[6] set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g -open in X
- 11) rg -closed[13] set g -closed[6] set if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open in X
- 12) Sg^* -closed if $cl(int(A) \subseteq U$, whenever $A \subseteq U$ and U is g -open in X

The complements of the above mentioned closed sets are their respective open sets

Definition:2.2A map $f:X \rightarrow Y$ is said to be

- (1) Continuous function if $f^{-1}(V)$ is closed in X for every closed set V in Y
- (2) b -continuous function if $f^{-1}(V)$ is b -closed in X for every closed set V in Y
- (3) g -continuous function if $f^{-1}(V)$ is g -closed in X for every closed set V in Y
- (4) α -continuous function if $f^{-1}(V)$ is α -closed in X for every closed set V in Y
- (5) w -continuous function if $f^{-1}(V)$ is w -closed in X for every closed set V in Y

- (6) g^* -continuous function if $f^{-1}(V)$ is g^* -closed in X for every closed set V in Y
- (7) Sg^* -continuous function if $f^{-1}(V)$ is Sg^* -closed in X for every closed set V in Y
- (8) Sg -continuous function if $f^{-1}(V)$ is Sg -closed in X for every closed set V in Y
- (9) gs -continuous function if $f^{-1}(V)$ is gs -closed in X for every closed set V in Y

Definition:2.3A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called

- (1) b -irresolute if $f^{-1}(V)$ is b -open set in (X, τ) for every b -open V in (Y, σ)
- (2) w -irresolute if $f^{-1}(V)$ is w -open set in (X, τ) for every w -open V in (Y, σ)
- (3) g -irresolute if $f^{-1}(V)$ is g -open set in (X, τ) for every g -open V in (Y, σ)
- (4) g^* -irresolute if $f^{-1}(V)$ is g^* -open set in (X, τ) for every g^* -open V in (Y, σ)
- (5) Sg^* -irresolute if $f^{-1}(V)$ is Sg^* -open set in (X, τ) for every Sg^* -open V in (Y, σ)

3.Sg*- CONTINUOUS FUNCTIONS AND Sg*-IRRESOLUTE FUNCTIONS

Definition:3.1Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be Strongly b star (briefly Sb^*) continuous function if $f^{-1}(V)$ is Sg^* -closed set in (X, τ) for every closed set V in (Y, σ)

Theorem:3.2Every closed map is Sg^* -closed map but not conversely

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a closed map . Then F be a closed set in (X, τ) . So $f(F)$ be a closed set in (Y, σ) . Now by the definition of Sg^* -closed, $f(F)$ is a Sg^* -closed set in (Y, σ) . We Know that closed $\Rightarrow Sg^*$ -closed [14] . f is Sg^* -closed map .

The converse of the above theorem need not be true as seen from the following example

Example:3.3Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$, $\sigma =\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Define a function $f:X \rightarrow Y$ as $f(a)=a$, $f(b)=f(c)=d, f(d)=b$. Let $A=\{a,b\}$ be a Sb^* -closed . $f(\{a,b\})=\{a,d\}$ which is not closed map

Remark:3.4Similarly we prove that the below theorems follows from the definition

- (i) Every θ -closed map is Sg^* -closed map
- (ii) Every δ -closed map is Sg^* -closed map

The converse of the above theorems need not be true as seen from the following examples .

Example:3.5 (i) Let $X=\{a,b,c,d\}$, $\tau =\{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma =\{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$ Let $f:X \rightarrow Y$ be an identity map Let $A=\{a,b\}$ be a Sb^* -closed . But $f(\{a,b\})=\{a,b\}$ which is not θ -closed map

(ii)Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma =\{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$ Define a $f:X \rightarrow Y$ as $f(a)=f(b)=a$ and $f(c)=f(d)=b$. Let $A=\{a,b,c\}$ be a Sg^* -closed . But $f(\{a,b,c\})=\{a,b\}$ which is not δ -closed map.

Theorem:3.6 Every Sg^* -closed map is g^* -closed map

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a Sg^* -closed map . Then F be a closed set in (X, τ) . So $f(F)$ be a Sg^* -closed set in (Y, σ) . Now by the definition of g^* -closed, $f(F)$ is a g^* -closed set in (Y, σ) . We Know that Sg^* -closed $\Rightarrow g^*$ -closed [2] . f is g^* -closed map .

The converse of the above theorem need not be true as seen from the following example

Example:3.7 Let $X=Y=\{a,b,c,d\}$, $\tau = \{X, \phi, \{a\}, \{a,c\}, \{a,c,d\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Let f be an identity map such that $f:X \rightarrow Y$. Let $A=\{c,d\}$ be a g^* -closed . $f(\{c,d\})=\{c,d\}$ which is not Sb^* -closed map

Remark:3.8 Similarly we prove that the below theorems follows from the definition

- (i) Every Sg^* -closed map is gs -closed map
- (ii) Every Sg^* -closed map is sg -closed map.

The converse of the above theorems need not be true as seen from the following examples .

Example:3.9(i) Let $X=Y=\{a,b,c,d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma = \{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$ Let $f:X \rightarrow Y$ be an identity map . Let $A=\{c\}$ be a gs -closed which is not Sg^* -closed map

(ii) Let $X=Y=\{a,b,c,d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma = \{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$. Let $f:X \rightarrow Y$ be an identity map Let $A=\{c,d\}$ be a sg -closed which is not Sg^* -closed map.

Theorem:3.10 Every continuous function is Sg^* -continuous map but not conversely

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a continuous function . Then by definition of continuous function , Let V be a closed set in (Y, σ) . Then $f^{-1}(V)$ is closed set in (X, τ) . Since, We Know that closed $\Rightarrow Sg^*$ -closed [14] . Then $f^{-1}(V)$ is Sg^* -closed set in (X, τ) . Therefore f is Sg^* -continuous function

Theorem:3.11 Every Sg^* -continuous function is g^* -continuous map but not conversely

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a Sg^* -continuous function . Then by definition of Sg^* -continuous function , Let V be a closed set in (Y, σ) . Then $f^{-1}(V)$ is Sg^* -closed set in (X, τ) . Since, We Know that $Sg^* \Rightarrow g^*$ -closed [14] . Then $f^{-1}(V)$ is g^* -closed set in (X, τ) . Therefore f is g^* -continuous function

Theorem:3.12 Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a map . Then the following statements are equivalent
 f is Sg^* - continuous

- (a) The inverse image of each open set in Y is Sg^* -open in X

Proof: Assume that $f:(X, \tau) \rightarrow (Y, \sigma)$ is Sg^* -continuous. Let G be an open set of (Y, σ) . Then G^c is closed in (Y, σ) . Since f is Sg^* -continuous, $f^{-1}(G^c)$ is Sg^* -closed in (X, τ) . But $f^{-1}(G^c) = X - f^{-1}(G)$. Thus $f^{-1}(G)$ is Sg^* -open in (X, τ) . Conversely, assume that the inverse image of each open set in (Y, σ) is Sg^* -open in (X, τ) . Let F be any closed set in (Y, σ) . By assumption F is Sg^* -open in (X, τ) . But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is Sg^* -open in (X, τ) and so $f^{-1}(F)$ is Sg^* -closed in (X, τ) . Therefore f is Sg^* -continuous. Hence (a) & (b) are equivalent.

Definition:3.13 A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called Sg^* -irresolute function if the inverse image of every Sg^* -closed in (Y, σ) is Sg^* -closed in (X, τ)

Theorem:3.14 If a function $f:(X, \tau) \rightarrow (Y, \sigma)$ is Sg^* -irresolute then it is Sg^* -continuous

Proof: Let F be any closed in (Y, σ) . Then F is Sg^* -closed set in (Y, σ) . As f is Sg^* -irresolute, $f^{-1}(F)$ is Sg^* -closed set in (X, τ) . Therefore f is Sg^* -continuous function

Theorem:3.15 Let $f:(X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \xi)$ be two functions. Then $g \circ f : (X, \tau) \rightarrow (Z, \xi)$ is Sg^* -continuous if f is Sg^* -continuous and g is continuous

Proof: Let Q be any closed set in (Z, ξ) . Then $g^{-1}(Q)$ is closed in (Y, σ) . Since g is a continuous. Sg^* -continuity of f implies $f^{-1}(g^{-1}(Q))$ is Sg^* -closed in (X, τ) . That is $(g \circ f)^{-1}(Q)$ is Sg^* -closed in (X, τ) . Hence $g \circ f$ is Sg^* -continuous.

Theorem:3.16 Let $f:(X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \xi)$ be two functions. Then $g \circ f : (X, \tau) \rightarrow (Z, \xi)$ is Sg^* -irresolute if f is Sg^* -irresolute and g is Sg^* -irresolute

Proof: Let Q be any closed set in (Z, ξ) . Since g is Sg^* -irresolute, $g^{-1}(Q)$ is Sg^* -irresolute in (Y, σ) . As f is Sg^* -irresolute $f^{-1}(g^{-1}(Q)) = (g \circ f)^{-1}(Q)$ is Sg^* -closed in (X, τ) . Hence $g \circ f$ is Sg^* -irresolute.

Theorem:3.17 Let $f:(X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \xi)$ be two functions. Then $g \circ f : (X, \tau) \rightarrow (Z, \xi)$ is Sg^* -continuous if f is Sg^* -irresolute and g is Sg^* -continuous

Proof: Let Q be any closed set in (Z, ξ) . Since g is Sg^* -continuous, $g^{-1}(Q)$ is Sg^* -closed in (Y, σ) . As f is Sg^* -irresolute $f^{-1}(g^{-1}(Q)) = (g \circ f)^{-1}(Q)$ is Sg^* -closed in (X, τ) . Hence $g \circ f$ is Sg^* -continuous.

Definition :3.18 For a subset A of a space $Sg^*\text{-cl}(A) = \bigcap \{F: A \subseteq F, F \text{ is } Sg^*\text{-closed in } X\}$ is called the Sg^* -closure of A

Definition :3.19 Let (X, τ) be a topological space and $\tau_{Sg^*} = \{V \subseteq X : Sg^*\text{-cl}(X-V) = X-V\}$

Theorem:3.20 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is Sg^* -continuous and U is open subset of X , then the restriction $f/U: U \rightarrow Y$ is g^* -continuous

Proof: Let V be any S^* -closed in (Y, σ) . Since f is any Sg^* -continuous then $f^{-1}(V)$ is open in (X, τ) . Since U is open in (X, τ) . $(f/U)^{-1}(V) = U \cap f^{-1}(V)$ is open in U . Hence f/U is Sg^* -continuous

Theorem 3.21: Let A be a subset of a topological space X . Then $x \in Sg^*\text{cl}(A)$ if and only if for any Sg^* -open set U containing x , $A \cap U \neq \emptyset$.

Proof: Let $x \in Sg^*\text{cl}(A)$ and suppose that, there is a Sg^* -open set U in X such that $x \in U$ and $A \cap U = \emptyset$ implies that $A \subset U^c$ which is Sg^* -closed in X implies $Sg^*\text{cl}(A) \subseteq Sg^*\text{cl}(U^c) = U^c$. since $x \in U$ implies that $x \notin U^c$ implies that $x \notin Sg^*\text{cl}(A)$, this is a contradiction.

Conversely, Suppose that, for any Sg^* -open set U containing x , $A \cap U \neq \emptyset$. To prove that $x \in Sg^*\text{cl}(A)$. Suppose that $x \notin Sg^*\text{cl}(A)$, then there is a Sg^* -closed set F in X such that $x \notin F$ and $A \subseteq F$. Since $x \notin F$ implies that $x \in F^c$ which is Sg^* -open in X . Since $A \subseteq F$ implies that $A \cap F^c = \emptyset$, this is a contradiction. Thus $x \in Sg^*\text{cl}(A)$.

Theorem 3.22: Let $f: X \rightarrow Y$ be a function from a topological space X into a topological space Y . If $f: X \rightarrow Y$ is Sg^* -continuous, then $f(Sg^*\text{cl}(A)) \subseteq \text{cl}(f(A))$ for every subset A of X .

Proof: Since $f(A) \subseteq \text{cl}(f(A))$ implies that $A \subseteq f^{-1}(\text{cl}(f(A)))$. Since $\text{cl}(f(A))$ is a closed set in Y and f is Sg^* -continuous, then by definition $f^{-1}(\text{cl}(f(A)))$ is a Sg^* -closed set in X containing A . Hence $Sg^*\text{cl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(Sg^*\text{cl}(A)) \subseteq \text{cl}(f(A))$.

Theorem 3.23: Let $f: X \rightarrow Y$ be a function from a topological space X into a topological space Y .

Then the following statements are equivalent:

(i) For each point x in X and each open set V in Y with $f(x) \in V$, there is a Sg^* -open set U in X such that $x \in U$ and $f(U) \subseteq V$.

(ii) For each subset A of X , $f(Sg^*\text{cl}(A)) \subseteq \text{cl}(f(A))$.

(iii) For each subset B of Y , $Sg^*\text{cl}(f^{-1}(B)) \subseteq f^{-1}(B)$.

Proof: (i) \rightarrow (ii) Suppose that (i) holds and let $y \in f(Sg^*\text{cl}(A))$ and let V be any open neighborhood of y . Since $y \in f(Sg^*\text{cl}(A))$ implies that there exists $x \in Sg^*\text{cl}(A)$ such that $f(x) = y$. Since $f(x) \in V$, then by (i) there exists a Sg^* -open set U in X such that $x \in U$ and $f(U) \subseteq V$. Since $x \in f(Sg^*\text{cl}(A))$, then by theorem 3.25 $U \cap A \neq \emptyset$. Therefore we have $y = f(x)$

$\in \text{cl}(f(A))$. Hence $f(\text{Sg}^*\text{cl}(A)) \subseteq \text{cl}(f(A))$.

(ii) \rightarrow (i) Let if (ii) holds and let $x \in X$ and V be any pen set in Y containing $f(x)$. Let $A = f^{-1}(V^c)$ this implies that $x \notin A$. Since $f(\text{Sg}^*\text{cl}(A)) \subseteq \text{cl}(f(A)) \subseteq V^c$ this implies that $\text{Sg}^*\text{cl}(A) \subseteq f^{-1}(V) = A$. Since $x \notin A$ implies that $x \notin \text{Sg}^*\text{cl}(A)$ and by theorem 3.25 there exists a Sg^* -open set U containing x such that $U \cap A \neq \emptyset$ and hence $f(U) \subseteq f(A^c) \subseteq V$.

(ii) \rightarrow (iii) Suppose that (ii) holds and Let B be any subset of Y . Replacing A by $f^{-1}(B)$ we get from (ii) $f(\text{Sg}^*\text{cl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$. Hence $\text{Sg}^*\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

(iii) \rightarrow (ii) Suppose that (iii) holds, let $B = f(A)$ where A is a subset of X . Then we get from (iii) $\text{Sg}^*\text{cl}(A) \subseteq \text{Sg}^*\text{cl}(f^{-1}(f(A))) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(\text{Sg}^*\text{cl}(A)) \subseteq \text{cl}(f(A))$.

Theorem 3.24: If a function $f: X \rightarrow Y$ is Sg^* -continuous, then $f(\text{Sg}^*\text{-cl}(A)) \subseteq \text{cl}(f(A))$ for every subset A of X .

Proof: Let $f: X \rightarrow Y$ be Sg^* -continuous. Let $A \subseteq X$. Then $\text{cl}(f(A))$ is closed in Y . Since f is Sg^* -continuous, $f^{-1}(\text{cl}(f(A)))$ is Sg^* -closed in X and $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\text{cl}(f(A)))$, implies $\text{Sg}^*\text{-cl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$. Hence $f(\text{Sg}^*\text{-cl}(A)) \subseteq \text{cl}(f(A))$.

Theorem 3.25: Let $f: X \rightarrow Y$ be a function. Let (X, τ) and (Y, σ) be any two spaces such that τ_{Sg^*} is a topology on X . Then the following statements are equivalent:

(i) For every subset A of X , $f(\text{Sg}^*\text{-cl}(A)) \subseteq \text{cl}(f(A))$ holds,

(ii) $f: (X, \tau_{\text{Sg}^*}) \rightarrow (Y, \sigma)$ is continuous.

Proof: Suppose (i) holds. Let A be closed in Y . By hypothesis $f(\text{Sg}^*\text{-cl}(f^{-1}(A))) \subseteq \text{cl}(f(f^{-1}(A))) \subseteq \text{cl}(f(A)) = A$. i.e., $\text{Sg}^*\text{-cl}(f^{-1}(A)) \subseteq f^{-1}(A)$. Also $f^{-1}(A) \subseteq \text{Sg}^*\text{-cl}(f^{-1}(A))$. Hence, $\text{Sg}^*\text{-cl}(f^{-1}(A)) = f^{-1}(A)$. This implies $(f^{-1}(A))^c \in \tau_{\text{Sg}^*}$. Thus $f^{-1}(A)$ is closed in (X, τ_{Sg^*}) and so f is continuous. This proves (ii).

Suppose (ii) holds. For every subset A of X , $\text{cl}(f(A))$ is closed in Y . Since $f: (X, \tau_{\text{Sg}^*}) \rightarrow (Y, \sigma)$ is continuous, $f^{-1}(\text{cl}(f(A)))$ is closed in (X, τ_{Sg^*}) that implies by Definition 3.23 „ $\text{Sg}^*\text{-cl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Now we have, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\text{cl}(f(A)))$. $\text{Sg}^*\text{-cl}(A) \subseteq \text{Sg}^*\text{-cl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Therefore $f(\text{Sg}^*\text{-cl}(A)) \subseteq \text{cl}(f(A))$.

Theorem 3.26: Let $f: X \rightarrow Y$ is Sg^* -continuous function and $g: Y \rightarrow Z$ is continuous function then $g \circ f: X \rightarrow Z$ is Sg^* -continuous. **Proof:** Let g be a continuous function and V be any open set in Z then $f^{-1}(V)$ is open in Y . Since f is Sg^* -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is Sg^* -open in X . Hence $g \circ f$ is Sg^* -continuous.

Theorem 3.27: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Let $h = g \circ f$. Then:

(i) h is Sg^* -continuous if f is Sg^* -irresolute and g is Sg^* -continuous and

(ii) h is Sg^* -continuous if g is continuous and f is Sg^* -continuous.

Proof: Let V be closed in Z . (i) Suppose f is Sg^* -irresolute and g is Sg^* -continuous. Since g continuous is Sg^* -continuous. $g^{-1}(V)$ is Sg^* -closed in Y . Since f is gr^* -irresolute, using the Definition $f^{-1}(g^{-1}(V))$ is Sg^* -closed in X . This proves (i).

(ii) Let g be continuous and f be Sg^* -continuous. Then $g^{-1}(V)$ is closed in Y . Since f is Sg^* -continuous, using the Definition $f^{-1}(g^{-1}(V))$ is Sg^* -closed in X . This proves (iii).

Theorem 3.28: A function $f: X \rightarrow Y$ be a bijection. Then the following are equivalent:

(i) f is Sg^* -open,

(ii) f is Sg^* -closed,

(iii) f^{-1} is Sg^* -irresolute.

Proof: Suppose f is Sg^* -open. Let F be Sg^* -closed in X . Then $X \setminus F$ is gr^* -open. By Definition $f(X \setminus F)$ is gr^* -open. Since f is bijection, $Y \setminus f(F)$ is gr^* -open in Y . Therefore f is Sg^* -closed. This proves (i) \Rightarrow (ii).

Let $g = f^{-1}$. Suppose f is Sg^* -closed. Let V be Sg^* -open in X . Then $X \setminus V$ is Sg^* -closed in X . Since f is Sg^* -closed, $f(X \setminus V)$ is Sg^* -closed. Since f is a bijection, $Y \setminus f(V)$ is Sg^* -closed that implies $f(V)$ is Sg^* -open in Y . since g and f are bijection $g^{-1}(V) = f(V)$ so that $g^{-1}(V)$ is Sg^* -open in Y . Therefore f^{-1} is Sg^* -irresolute. This proves (ii) \Rightarrow (iii).

Suppose f^{-1} is Sg^* -irresolute. Let V be Sg^* -open in X . Then $X \setminus V$ is Sg^* -closed in X . Since f^{-1} is Sg^* -irresolute and $(f^{-1})^{-1}(X \setminus V) = f(X \setminus V) = Y \setminus f(V)$ is Sg^* -closed in Y that implies $f(V)$ is Sg^* -open in Y . Therefore f is Sg^* -open. This proves (iii) \Rightarrow (i).

Theorem 3.29 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are gr^* -irresolute, then $g \circ f: X \rightarrow Z$ is Sg^* -irresolute.

Proof: Let g be an Sg^* -irresolute function and V be any Sg^* -open in Z , then $f^{-1}(V)$ is Sg^* -open set in Y , since f is gr^* -irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is Sg^* -open in (X, τ) . Hence $g \circ f$ is Sg^* -irresolute.

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