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Research Article

ON TERNARY QUADRATIC DIOPHANTINE EQUATION x^2-xy+y^2 = 12z^2

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ABSTRACT

The Homogenous Ternary Quadratic Diophantine Equation is given by $x^2 - xy + y^2 = 12z^2$ and analyzed for its patterns of non zero distinct integer solutions. Introducing the linear transformation x= u+ v, y=u-v and employing the method of factorization, different patterns of non zero distinct integer solutions to the above equation are obtained. A few of the interesting relationships between the solution and polygonal numbers are obtained.

Keywords: Homogenous Quadratic, Ternary Quadratic, Integer solutions.

MSC2010: 11D09

1. INTRODUCTION

The ternary quadratic Diophantine equation offers an unlimited field for research because of their variety [1-2]. In particular, one may refer [3-16] for finding integer points on the some specific three dimensional surface. This communication concern with yet another ternary quadratic equation $x^2 - xy + y^2 = 12z^2$ representing cone for determining its infinitely many integral solutions. Employing the integral solutions on the given cone, a few interesting relations among the special polygonal numbers are given.

NOTATION USED

$t_{m,n} = n\left(1+\frac{(n-1)(m-2)}{2}\right)$ =Polygonal number of rank n with sides m.

2. MEHTOD OF ANALYSIS

Consider the equation

$$x^2 - xy + y^2 = 12z^2 \quad (1)$$

The substitution of linear transformations

$$x = u+v \text{ and } x = u-v \quad (u \neq v \neq 0) \quad (2)$$

In (1) leads to

$$u^2 + 3v^2 = 12z^2 \quad (3)$$

The above equation is solved through different methods and using (2), different patterns of integer solution to (1) are obtained.

2.1. PATTERN

Write 12 as

$$12 = (3+i\sqrt{3})(3-i\sqrt{3}) \quad (4)$$

Assume $z = a^2 + 3b^2$ where a, b>0 (5)

Using (4) and (5) in (3), and applying the method of factorization, define.

$$(u+i\sqrt{3}v) = (3+i\sqrt{3})(a+i\sqrt{3}b)^2 \quad (6)$$

Equating the real and imaginary parts, we have

$$u = u(a,b) = 3a^2 - 9b^2 - 6ab$$

$$v = v(a,b) = a^2 - 3b^2 + 6ab$$

Substituting the above u and v in equation (2), the value of x and y are given by

$$\left.\begin{aligned} x &= x(a,b) = 4a^2 - 12b^2 \\ y &= y(a,b) = 2a^2 - 6b^2 - 12ab \end{aligned}\right\} \quad (7)$$

Thus (5) and (7) represent non-zero distinct integral solution of (1) in two parameters.

PROPERTIES

1. $z(a+1,a) - t_{6,a} - t_{6,a} \equiv 0 \pmod 1$
2. $y(3,b) - t_{10,a} - t_{6,a} \equiv 0 \pmod 2$
3. $x(a,1) + y(a,1) - t_{8,a} - t_{8,a} \equiv 0 \pmod 2$

4. $y(a,2) - t_{4,a} - t_{4,a} \equiv 0 \pmod 4$
5. $y(a,a+1) - t_{22,a} - t_{14,a} \equiv 0 \pmod 3$
6. $y(a,a+1) - t_{18,a} - t_{18,a} \equiv 0 \pmod 6$

2.2. PATTERN

Treating (1) as a quadratic in x and solving for x, we get

$$X = \frac{1}{2}\left[y \pm \sqrt{48z^2 - 3y^2}\right] \qquad (8)$$

Let $\alpha^2 = 48z^2 - 3y^2$

$= 3\ (4z)^2 + 3y^2$

$$\alpha^2 = 3[(4z + y)(4z - y)] \qquad (9)$$

Write (9) in the form of ratio as

$$\frac{4z + y}{\alpha} = \frac{\alpha}{3(4z - y)} = \frac{A}{B}$$

This is equivalent to the following two equations

$$\alpha B - 12Az + 3Ay = 0$$
$$\alpha A - 4Bz - By = 0$$

On employing the method of cross multiplication, we get

$$\left.\begin{array}{l} \alpha = 24AB \\ Y = 12A^2 - 4B^2 \end{array}\right\}$$

$$z = 3A^2 + B^2 \qquad (10)$$

Substituting the values of y and z from (10) in (8) , the non-zero distinct integer value of x, are given by

$$x = 6A^2 - 2B^2 \pm 12AB \qquad (11)$$

Thus (10) and (11) represent the non-zero distinct integer solution of equation (1) in two parameters.

SET I:

$x = 6A^2 - 2B^2 + 12AB$

$y = 12A^2 - 4B^2$

$z = 3A^2 + B^2$

SET II:

x = $6A^2 - 2B^2 - 12AB$

$y = 12A^2 - 4B^2$

$z = 3A^2 + B^2$

PROPERTIES

1. $x(A,2) - t_{10,A} - t_{6,A} \equiv 0 (\bmod 4)$
2. $x(1,B) + Y(1,B) - t_{8,B} - t_{8,B} \equiv 0 (\bmod 2)$
3. $z(B+1, B+1) - t_{6,B} - t_{6,B} \equiv 0 (\bmod 2)$
4. $y(A+1.A+1) - t_{12,A} - t_{8,A} \equiv 0 (\bmod 4)$
5. $y(A, A+1) - t_{12,A} - t_{8,A} \equiv 0 (\bmod 2)$

2.2 Mathematical Model

Equation (9) Can also be expressed in the form of ratio in three different ways as follows:

1. $\frac{3(4z-y)}{\alpha} = \frac{\alpha}{4z+y} = \frac{A}{B}$
2. $\frac{3(4z+y)}{\alpha} = \frac{\alpha}{4z-y} = \frac{A}{B}$
3. $\frac{4z-y}{\alpha} = \frac{\alpha}{3(4z+y)} = \frac{A}{B}$

Repeating the analysis as above, we get different pattern of solution to (1).

2.3 PATTERN

Rewrite (3) as

$$3v^2 = 12z^2 - u^2 \qquad (12)$$

Write 3 as, $3 = (2\sqrt{3}+3)(2\sqrt{3}-3)$ (13)

Let $v = 12a^2 - b^2$, a, b $\neq 0$ (14)

Using (13) and (14) in (12) and employing the method of factorization, we write

$$2\sqrt{3}z + u = (2\sqrt{3}+3)(2\sqrt{3}a + b)^2$$

Equating the rational and irrational parts, we have

$$z = z(a,b) = 12a^2 + b^2 + 6ab \tag{15}$$

$$u = u(a,b) = 36a^2 + 3b^2 + 24ab \tag{16}$$

Satisfying (14) and (16) in (2) , the value of x and y are

$$\left.\begin{aligned} x = x(a,b) = 48a^2 + 2b^2 + 24ab \\ y = y(a,b) = 24a^2 + 4b^2 + 24ab \end{aligned}\right\} \tag{17}$$

Thus (17) and (15) represent the integer solution to (1)

Properties

1. $Z(a,1) - t_{18,a} - t_{10,a} \equiv 0(\text{mod}\,1)$
2. $y(1,2b) + (1,2b) - t_{32,b} - t_{12,b} \equiv 0(\text{mod}\,3)$
3. $y(a,1) - t_{24,b} - t_{28,b} \equiv 0(\text{mod}\,3)$
4. $x(1,a) - t_{4,b} - t_{4,b} \equiv 0(\text{mod}\,3)$

CONCLUSION

In this paper ,we have presented different pattern of integer solutions to the ternary quadratic equation $x^2 - xy + y^2 = 12z^2$ representing the cone .As this Diophantine equations are rich in variety , one may attempt to find integer solutions to other choices of equations along with suitable properties.

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