International Journal of Research Instinct
(Peer-reviewed Journal)

ISSN ONLINE 2348-2095
Research Article

# INTERVAL VALUED INTUITIONISTIC FUZZY GROUPS AND ITS LEVEL SUBGROUPS

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## ABSTRACT

In this article, we define the algebraic structures of interval valued intuitionistic fuzzy subgroups and some related properties that are investigated. The purpose of this study is to implement the fuzzy set theory and group theory in interval valued intuitionist fuzzy subgroups and Characterizations of interval valued intuitionistic fuzzy level subsets of a interval valued intuitionistic fuzzy subgroups of a group.

**Keywords**— Fuzzy set, multi-fuzzy set, fuzzy subgroup, multi-fuzzy subgroup, anti-fuzzy subgroup, multi-anti fuzzy subgroup, interval valued intuitionist fuzz y subset, interval valued intuitionistic fuzzy subgroups, interval valued intuitionistic fuzzy level subsets, interval valued intuitionistic fuzzy level subgroups

## 1.INTRODUCTION

After the introduction of the concept of fuzzy sets by L.A.Zadeh [1], researchers were conducted the generalizations of the notion of fuzzy sets, A. Rosenfeld [2] introduced the concept of fuzzy group and the idea of "intuitionistic fuzzy set" was first published by K.T. Atanassov [3,7]. W.D.Blizard [4] introduced the concept of fuzzy multi-set theory.Also Shinoj. T.K and Sunil Jacob [5] produced some results in Intuitionistic Fuzzy Multi-sets.P.K.Sharma discuss $(\alpha, \beta)$-cut of Intuitionistic Fuzzy groups in 2014[6].In this chapter we define *interval valued intuitionistic* fuzzy sets and *interval valued intuitionistic* fuzzy subgroups and some of their properties

## 2. PRELIMINARIES

### 2.1 Definition: Fuzzy Set

Let $X$ be a non-empty set. A fuzzy set $A$ on $X$ is a mapping $A: X \to [0,1]$ and is defined as $A = \{x \in X / (x, \mu(x))\}$

### 2.2. Definition: Image

Let $X$ and $Y$ be any two sets.Let $f: X \to Y$ be afunction. If $\mu$ is a fuzzy set on $X$ then the image $\mu$ under $f$ is a fuzzy set on $Y$ and is defined by

$f(\mu)(y) = v(y) = \sup_{x \in f^{-1}(y)} \mu(x), \ \forall\, y \in Y$ is called image of $\mu$ under $f$

### 2.3. Definition: Pre-image

Let $X$ and $Y$ be any two sets. Let $f: X \to Y$ b a function. If $S$ is a fuzzy set on $Y$ then the preimage of $S$ under $f$ is a fuzzy set on $X$ & is defined by

$$\left(f^{-1}(S)\right)(x) = S\left(f(x)\right)$$

### 2.4. Definition: Level Fuzzy Subset

Let $A$ be a fuzzy subset of a set $X$. For $t \in [0, 1]$, $A_t = \{x \in X / A(x) \geq t\}$ is called a level fuzzy subset of $A$

### 2.5. Definition: Fuzzy Multi set

Let $X$ be a non-empty set. A Fuzzy Multiset (FMS) $A$ drawn from $X$ is characterized by a function ' Count membership' of $A$ denotred by $CM_A$ such that $CM_A : X \to Q$ where $Q$ is the set of all crisp finiteset drawn from the unit interval $[0,1]$.Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0,1]$. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_{A}(x)$. It is denoted by $\left(\mu_{A_1}(x), \mu_{A_2}(x), ........\mu_{A_k}(x)\right)$ where $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq ........ \geq \mu_{A_k}(x)$

$A = \left\{\left\langle x : \left(\mu_{A_1}(x), \mu_{A_2}(x), ........\mu_{A_k}(x)\right)\right\rangle : x \in X\right\}$

**2.6. Definition: Intuitionistic Fuzzy set (IFS)**

Let $X$ be a non empty set. An Intuitionistic Fuzzy set $A$ on $X$ is an object having the form

$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0,1]$ & $\gamma_A : X \to [0,1]$ are the degree of membership and non- membership functions respectively with $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$

**2.7. Definition: Interval Valued Fuzzy Set (IVFS)**

Let $[I]$ be the set of all closed subintervals of the interval $[0,1]$ and $M = [M_L, M_U] \in [I]$ where $M_L$ and $M_U$ are the lower extreme and upper extreme, respectively. For a set $X$, an *IVFS* $A$ on $X$ is given by

$$A = \{\langle x, M_A(x)\rangle / x \in X\}$$

Where the function $M_A : X \to [0,1]$ defines the degree of membership of an element $x$ to $A$, and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ called an interval- valued fuzzy number

**2.8. Definition: Interval Valued Intuitionistic Fuzzy Set (IVIFS):**

For a set $X$, an IVIFS $A$ on $X$ is an object having the form $A = \{\langle x, M_A(x), N_A(x)\rangle / x \in X\}$ where $M_A : X \to [I]$ and $N_A : X \to [I]$ represents the degree of membership and non- membership, $0 \leq Sup(M_A(x)) + Sup(N_A(x)) \leq 1$, for every $x \in X$. $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$

Hence $A = \{[M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\}$

**2.9.Properties of IVIFS**

Let $A = \{\langle x, M_A(x), N_A(x)\rangle / x \in X\}$ and $B = \{\langle x, M_B(x), N_B(x)\rangle / x \in X\}$ be any two IVIFS of X, then

(1). $A \subseteq B \Leftrightarrow M_A(x) \le M_B(x)$ & $N_A(x) \ge N_B(x) \quad \forall\ x, y \in X$

(2). $A = B \Leftrightarrow M_A(x) = M_B(x)$ & $N_A(x) = N_B(x) \quad \forall\ x, y \in X$

(3). $A \cap B = \left\{ \left\langle x, \left(M_A \cap M_B\right)(x), \left(N_A \cap N_B\right)(x) \right\rangle : x \in X \right\}$ where

$$\left(M_A \cap M_B\right)(x) = \text{Min}\left\{M_A(x), M_B(x)\right\} = M_A(x) \wedge M_B(x)$$

$$\left(N_A \cap N_B\right)(x) = \text{Max}\left\{N_A(x), N_B(x)\right\} = N_A(x) \vee N_B(x)$$

(4). $A \cup B = \left\{ \left\langle x, \left(M_A \cup M_B\right)(x), \left(N_A \cup N_B\right)(x) \right\rangle : x \in X \right\}$ where

$$\left(M_A \cup M_B\right)(x) = \text{Max}\left\{M_A(x), M_B(x)\right\} = M_A(x) \vee M_B(x)$$

$$\left(N_A \cup N_B\right)(x) = \text{Min}\left\{N_A(x), N_B(x)\right\} = N_A(x) \wedge N_B(x)$$

(5). If $A = \left\{ \left\langle x, M_A(x), N_A(x) \right\rangle : x \in X \right\}$, then

$$\left(A\right)^c = A = \left\{ \left\langle x, 1 - M_A(x), 1 - N_A(x) \right\rangle : x \in X \right\}$$

**2.10. Definition: Fuzzy subgroup**

Let $G$ be a group. A fuzzy subset $A$ of $G$ is said to be a **fuzzy subgroup** of $G$ if

$(i). A(xy) \ge \text{Min}\left\{A(x), A(y)\right\}$

$(ii). A(x^{-1}) \ge A(x) \quad \forall\ x, y \in G$

**2.11. Definition: Anti Fuzzy subgroup**

Let $G$ be a group. A fuzzy subset $A$ of $G$ is said to be an anti-fuzzy subgroup of $G$ if

$(i). A(xy) \le \max\left\{A(x), A(y)\right\}$

$(ii). A(x^{-1}) = A(x) \quad \forall\ x, y \in G$

**2.12. Definition: Multi Fuzzy subgroup**

Let $G$ be a group. A multi-fuzzy subset $A$ of $G$ is said to be an multi-fuzzy subgroup of

$G$ if

$(i).A(xy) \geq \min\{A(x), A(y)\}$

$(ii).A(x^{-1}) \geq A(x) \quad \forall\, x, y \in G$

**2.13. Definition: Multi –Anti Fuzzy subgroup**

Let $G$ be a group. A multi-fuzzy subset $A$ of $G$ is said to bean multi-anti-fuzzy subgroup of $G$ if

$(i).A(xy) \leq \mathrm{Max}\{A(x), A(y)\}$

$(ii).A(x^{-1}) \geq A(x) \quad \forall\, x, y \in G$

**2.14. Definition: Intuitionistic Fuzzy subgroup**

An IFS $A = \{(x, \mu(x), \nu(x)) : x \in X\}$ of a group $G$ is said to be **intuitionistic fuzzy subgroup** of $G$ (In short IFSG) if

(i). $\mu(xy) \geq \mu(x) \wedge \mu(y)$

(ii). $\mu(x^{-1}) = \mu(x)$

(iii). $\nu(xy) \leq \nu(x) \vee \nu(y)$

(iv). $\nu(x^{-1}) = \nu(x)$, for all $x, y \in G$

**2.15. Definition: Multi-Intuitionistic Fuzzy subgroup (In short MIFSG)**

An MIFS $A = \{(x, \mu_{A_i}(x), \nu_{A_i}(x)) : x \in X\}$ of a group $G$ is said to be **multi- intuitionistic fuzzy subgroup** of $G$ (In short MIFSG) if

(i). $\mu_{A_i}(xy) \geq \mu_{A_i}(x) \wedge \mu_{A_i}(y)$

(ii). $\mu_{A_i}(x^{-1}) = \mu_{A_i}(x)$

(iii). $\nu_{A_i}(xy) \leq \nu_{A_i}(x) \vee \nu_{A_i}(y)$

(iv). $\nu_{A_i}(x^{-1}) = \nu_{A_i}(x)$, for all $x, y \in G$

**2.16. Definition: Intuitionistic Fuzzy Normal Subgroup (IFNSG)**

An IFSG $A=\{(x,\mu(x),\nu(x)):x\in X\}$ of a group $G$ is said to be **intuitionistic fuzzy normal subgroup** of $G$ (IFNSG) if

(i). $\mu(xy)=\mu(yx)$

(ii). $\nu(xy)=\nu(yx)$ for all $x,y\in G$

**2.17. Definition: Interval Valued Intuitionistic Fuzzy Subgroup (In short IVIFSG)**

**An IVIFS $A=\{(x,M(x),N(x)):x\in X\}$ of a group $G$ is said to be interval valued intuitionistic fuzzy subgroup of $G$ (In short IVIFSG) if**

**(i).** $M(xy)\geq M(x)\wedge M(y)$

**(ii).** $M(x^{-1})=M(x)$

**(iii).** $N(xy)\leq N(x)\vee N(y)$

**(iv).** $N(x^{-1})=N(x),$ **for all** $x,y\in G$

**2.18. Definition: Interval Valued Intuitionistic Anti FuzzySubgroup (In short IVIAFSG**

An IVIAFS $A=\{(x,M(x),N(x)):x\in X\}$ of a group $G$ is said to be **interval valued intuitionistic anti fuzzy subgroup** of $G$ (In short IVIAFSG) if

(i). $M(xy)\leq M(x)\vee M(y)$

(ii). $M(x^{-1})=M(x)$

(iii). $N(xy)\geq N(x)\wedge N(y)$

(iv). $N(x^{-1})=N(x),$ for all $x,y\in G$

**Theorem : 3.1**

Let $A$ be IVIFSG of a group $G$ and $'e'$ is the identity element of $G$ then

$(i)$. $M(x)\leq M(e)$ & $N(x)\geq N(e)$

$(ii)$. The subset $H_1=\{x\in A/M(x)=M(e)\}$ is a subgroup of $G$

$(iii)$. The subset $H_2=\{x\in A/N(x)=N(e)\}$ is a subgroup of $G$

**Proof:**

Let $A$ be an IVIFSG of a group $G$ and $e\in G$

Let $x \in A \Rightarrow x \in G$

Now, $M(x) = \text{Min}\{M(x), M(x)\}$

$= \text{Min}\{M(x), M(x^{-1})\}$ as $A$ is a IVIFSG

$\leq M(xx^{-1}) = M(e)$ as $A$ is a IVIFSG

Hence $M(x) \leq M(e), \quad \forall x \in G$

Let $x \in G$

Now, $N(x) = \text{Max}\{N(x), N(x)\}$

$= \text{Max}\{N(x), N(x^{-1})\}$

$\geq N(xx^{-1}) = N(e)$

Hence $N(x) \geq N(e), \quad \forall x \in G$

$(ii)$. Let $H_1 = \{x \in A / M(x) = M(e)\}$

Clearly $H_1$ is non – empty

Let $x, y \in H_1 \Rightarrow M(x) = M(y) = M(e)$

Now, $M(xy^{-1}) \geq \text{Min}\{M(x), M(y^{-1})\}$

$= \text{Min}\{M(x), M(y)\}$

$= \text{Min}\{M(e), M(e)\}$

$= M(e)$

That is $M(xy^{-1}) \geq M(e)$ and obviously $M(xy^{-1}) \leq M(e)$

Hence $M(xy^{-1}) = M(e) \Rightarrow xy^{-1} \in H_1 \Rightarrow$ $H_1$ is a subgroup of G

$(ii)$. Let $H_2 = \{x \in A / N(x) = N(e)\}$

Clearly $H_2$ is non – empty

Let $x, y \in H_2 \Rightarrow N(x) = N(y) = N(e)$

Now, $N(xy^{-1}) \leq \text{Max}\{N(x), N(y^{-1})\}$

$= \text{Max}\{N(x), N(y)\}$

$= \text{Max}\{M(e), M(e)\}$

$= N(e)$

That is $N(xy^{-1}) \leq N(e)$ and obviously $N(xy^{-1}) \geq N(e)$

Hence $N(xy^{-1}) = N(e) \Rightarrow xy^{-1} \in H_2 \Rightarrow H_2$ is a subgroup of G

**Theorem : 3.2**

Let $A$ be a IVIFSG of a group $G$ iff $A^C$ is a interval valued intuitionistic anti-fuzzy subgroup of a group $G$

**Proof:**

Suppose $A$ is a *IVIFSG* of a group $G$

Then $\forall x, y \in G,\ M(xy) \geq \text{Min}\{M(x), M(y)\}$ & $N(xy) \leq \text{Max}\{N(x), N(y)\}$

$\Leftrightarrow\ 1 - M^C(xy) \geq \text{Min}\{1 - M^C(x), 1 - M^C(y)\}$ & $1 - N^C(xy) \leq \text{Max}\{1 - N^C(x), 1 - N^C(y)\}$

$\Leftrightarrow\ M^C(xy) \leq 1 - \text{Min}\{1 - M^C(x), 1 - M^C(y)\}$ & $N^C(xy) \geq 1 - \text{Max}\{1 - N^C(x), 1 - N^C(y)\}$

$\Leftrightarrow\ M^C(xy) \leq \text{Max}\{M^C(x), M^C(y)\}$ & $N^C(xy) \geq \text{Min}\{N^C(x), N^C(y)\}$

We have $M(x) = M(x^{-1})$ $\forall x \in G$ & We have $N(x) = N(x^{-1})$ $\forall x \in G$

$\Leftrightarrow 1 - M^C(x) = 1 - M^C(x^{-1})$ & $1 - N^C(x) = 1 - N^C(x^{-1})$

Hence $A^C$ is a interval valued intuitionistic anti-fuzzy subgroup of a group $G$

**Theorem : 3.3**

Let $A$ be a IVIF subgroup of a group $G$ and $'e'$ is the identity element of $G$ then

$M(xy^{-1}) = M(e) \Rightarrow M(x) = M(y)$ & $N(xy^{-1}) = N(e) \Rightarrow N(x) = N(y)$ $\forall x, y \in G$

**Proof:**

Let $A$ be a IVIF subgroup of a group $G$ and $'e'$ is the identity element of $G$ This implies

(i). $M(xy) \geq M(x) \wedge M(y)$

(ii). $M(x^{-1}) = M(x)$

(iii). $N(xy) \leq N(x) \vee N(y)$

(iv). $N(x^{-1}) = N(x)$, for all $x, y$ Î $G$

Let $M(xy^{-1}) = M(e) \quad \forall x, y \in G$

Now , $M(x) = M\left(x\left(y^{-1}y\right)\right)$

$= M\left(xy^{-1}(y)\right)$ as $G$ is a group, Associative property hold

$\geq \text{Min}\left\{M\left(xy^{-1}\right), M\left(y\right)\right\}$, as $A$ is a IVIFSG

$= \text{Min}\left\{M\left(e\right), M\left(y\right)\right\}$

$= M\left(y\right) \qquad \Rightarrow M(x) \geq M(y)$........(1)

Now, $M(y) = M(y^{-1})$

$= M(ey^{-1})$

$= M\left(\left(x^{-1}x\right)y^{-1}\right)$

$= M\left(\left(x^{-1}\right)xy^{-1}\right)$

$\geq \text{Min}\left\{M\left(x^{-1}\right), M\left(xy^{-1}\right)\right\}$

$= \text{Min}\left\{M\left(x^{-1}\right), M\left(e\right)\right\}$

$= \text{Min}\left\{M\left(x\right), M\left(e\right)\right\}$

$= M\left(x\right)$

Hence $M\left(y\right) \geq M\left(x\right)$............(2)

From (1) & (2), $M\left(x\right) = M\left(y\right), \quad \forall x, y \in G$

Similarly, Let $N(xy^{-1}) = N(e) \quad \forall x, y \in G$

Now , $N(x) = N\left(x\left(y^{-1}y\right)\right)$

$= N\left(xy^{-1}(y)\right)$

$\leq \text{Max}\left\{N\left(xy^{-1}\right), N\left(y\right)\right\}$

$= \text{Max}\{N(e), N(y)\}$

$= N(y) \qquad \Rightarrow N(x) \le N(y)$........(3)

Now, $N(y) = N(y^{-1})$

$= N(ey^{-1})$

$= N\left(\left(x^{-1}x\right)y^{-1}\right)$

$= N\left(\left(x^{-1}\right)xy^{-1}\right)$

$\le \text{Max}\left\{N\left(x^{-1}\right), N\left(xy^{-1}\right)\right\}$

$= \text{Max}\left\{N\left(x^{-1}\right), N(e)\right\}$

$= \text{Max}\{N(x), N(e)\}$

$= N(x)$

Hence $N(y) \le N(x)$............(4)

From (3) & (4), $N(x) = N(y), \quad \forall x, y \in G$

**Theorem : 3.4**

$A$ is an IVIFSG of a group $G$ if and only $M(xy^{-1}) \ge \min\{M(x), M(y)\}$ & $N(xy^{-1}) \le Max\{N(x), N(y)\} \quad \forall x, y \in G$

**Proof:**

Let $A$ be a IVIF subgroup of a group $G$ and $'e'$ is the identity element of $G$ This implies

(i). $M(xy) \ge M(x) \wedge M(y)$

(ii). $M(x^{-1}) = M(x)$

(iii). $N(xy) \le N(x) \vee N(y)$

(iv). $N(x^{-1}) = N(x)$, for all $x, y$ **Î** $G$

Now, $M(xy^{-1}) \ge \text{Min}\left\{M(x), M(y^{-1})\right\}$

$= \text{Min}\{M(x), M(y)\}$

$\Leftrightarrow M(xy^{-1}) \geq \text{Min}\{M(x), M(y)\}$

Similarly, Now, $N(xy^{-1}) \leq \text{Max}\{N(x), N(y^{-1})\}$

$= \text{Max}\{N(x), N(y)\}$

$\Leftrightarrow N(xy^{-1}) \leq \text{Max}\{N(x), N(y)\}$

**2.19. Definition: Interval Valued Intuitionistic Fuzzy Level subsets**

Let $A = \{(x, M_A(x), N_A(x)) / x \in X\}$ be a IVIFS on $X$. Then for $\alpha, \beta \in D[0,1]$, the set $A^{[\alpha,\beta]} = \{x \in X / M_A(x) \geq \alpha \text{ and } N_A(x) \leq \beta\}$ is called the $(\alpha, \beta)-$level subsets of $A$ .The sets is called the upper and lower level subsets of $A$, respectively. Clearly $A^{[\alpha,\beta]} = U(M_A, \alpha) \cap L(N_A, \beta)$. Also the set $A^{[\alpha,\beta]} = \{x \in X / M_A(x) > \alpha \text{ and } N_A(x) < \beta\}$ is called the strong level subset of $A$

**Proposition:3.2**

If $A$ and $B$ be two IVIF level sets of a universe set $X$ , then the following holds

(i). $A^{[\alpha,\beta]} \subseteq A^{[\delta,\theta]}$ if $\alpha \geq \delta$ and $\beta \leq \theta$

(ii). $A^{[1-\beta,\beta]} \subseteq A^{[\alpha,\beta]} \subseteq A^{[\alpha,1-\alpha]}$

(iii). $A \subseteq B \Rightarrow A^{[\alpha,\beta]} \subseteq B^{[\alpha,\beta]}$

(iv). $(A \cap B)^{[\alpha,\beta]} = A^{[\alpha,\beta]} \cap B^{[\alpha,\beta]}$

(v). $(A \cup B)^{[\alpha,\beta]} \supseteq A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]}$ equal if $\alpha + \beta = 1$

**Proof. (i).**

Let $x \in A^{[\alpha,\beta]} \Rightarrow M_A(x) \geq \alpha,\ N_A(x) \leq \beta$. ….(1)

Given $\delta \leq \alpha \leq M_A(x)$ and $\theta \geq \beta \geq N_A(x)$.

Hence $M_A(x) \geq \delta$ and $N_A(x) \leq \theta$

This implies $x \in A^{[\delta,\theta]}$ …… (2).

From (1) and (2) $A^{[\alpha,\beta]} \subseteq A^{[\delta,\theta]}$

(ii).Since $\alpha+\beta \le 1$, implies that $1-\beta \ge \alpha$ and $\beta \le \beta$

By part.(i). $A^{[1-\beta,\beta]} \subseteq A^{[\alpha,\beta]}$………. (3)

Again $\alpha+\beta \le 1$, implies that $\alpha \ge \alpha$ and $\beta \le 1-\alpha$ By part.(i). $A^{[\alpha,\beta]} \subseteq A^{[\alpha,1-\alpha]}$…(4)

From (3) and (4) $A^{[1-\beta,\beta]} \subseteq A^{[\alpha,\beta]} \subseteq A^{[\alpha,1-\alpha]}$

(iii). $x \in A^{[\alpha,\beta]} \Rightarrow M_A(x) \ge \alpha,\ N_A(x) \le \beta$

$A \subseteq B \Rightarrow M_B(x) \ge M_A(x) \ge \alpha$, and $N_B(x) \le N_A(x) \le \beta$
$\Rightarrow M_B(x) \ge \alpha$, and $N_B(x) \le \beta$ and so $x \in B^{[\alpha,\beta]}$ Hence $A^{[\alpha,\beta]} \subseteq B^{[\alpha,\beta]}$

(iv).Since $A \cap B \subseteq A$ and $A \cap B \subseteq A$

Therefore by part (i)

$(A\cap B)^{[\alpha,\beta]} \subseteq A^{[\alpha,\beta]}$ and $(A\cap B)^{[\alpha,\beta]} \subseteq B^{[\alpha,\beta]}$

$\Rightarrow (A\cap B)^{[\alpha,\beta]} \subseteq A^{[\alpha,\beta]} \cap B^{[\alpha,\beta]}$ ……… (5)

Let $x \in A^{[\alpha,\beta]} \cap B^{[\alpha,\beta]} \Rightarrow x \in A^{[\alpha,\beta]}$ and $x \in B^{[\alpha,\beta]}$
$\Rightarrow M_A(x) \ge \alpha$ & $N_A(x) \le \beta$ and $M_B(x) \ge \alpha$ & $N_B(x) \le \beta$
$\Rightarrow M_A(x) \ge \alpha$ & $M_B(x) \ge \alpha$ and $N_A(x) \le \beta$ & $N_B(x) \le \beta$

$\Rightarrow M_A(x) \wedge M_B(x) \ge \alpha$ and $N_A(x) \vee N_B(x) \le \beta$
$\Rightarrow \left(M_A \cap M_B\right)(x) \ge \alpha$ and $\left(N_A \cap N_B\right)(x) \le \beta$

$\Rightarrow x \in (A\cap B)^{[\alpha,\beta]}$ ………………..(6)

From (5) and (6) , $(A\cap B)^{[\alpha,\beta]} = A^{[\alpha,\beta]} \cap B^{[\alpha,\beta]}$

**(v).** since $A \subseteq A \cup B$ and $B \subseteq A \cup B$

Therefore by part (i)

$A^{[\alpha,\beta]} \subseteq (A\cup B)^{[\alpha,\beta]}$ and $B^{[\alpha,\beta]} \subseteq (A\cup B)^{[\alpha,\beta]}$

$\Rightarrow A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]} \subseteq (A \cup B)^{[\alpha,\beta]}$ ……… (7)

Let $x \in (A \cup B)^{[\alpha,\beta]} \Rightarrow (M_A \cup M_B)(x) \geq \alpha$ and $(N_A \cup N_B)(x) \leq \beta$

$\Rightarrow M_A(x) \geq \alpha$ & $N_A(x) \leq \beta$ and $M_B(x) \geq \alpha$ & $N_B(x) \leq \beta$

$\Rightarrow M_A(x) \vee M_B(x) \geq \alpha$ and $N_A(x) \wedge N_B(x) \leq \beta$

If $M_A(x) \geq \alpha,$ then $N_A(x) \leq 1 - M_A(x) \leq 1 - \alpha = \beta$

$\Rightarrow x \in A^{[\alpha,\beta]} \subseteq A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]}$

Similarly if $M_B(x) \geq \alpha,$ then $N_B(x) \leq 1 - M_B(x) \leq 1 - \alpha = \beta$

$\Rightarrow x \in B^{[\alpha,\beta]} \subseteq A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]}$

We see that $x \in (A \cup B)^{[\alpha,\beta]} \Rightarrow x \in A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]}$

$\Rightarrow (A \cup B)^{[\alpha,\beta]} \subseteq A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]}$…………..(8)

From (7) and (8), $(A \cup B)^{[\alpha,\beta]} \supseteq A^{[\alpha,\beta]} \cup B^{[\alpha,\beta]}$

**Theorem: 3.5**

If $A$ is Interval valued intuitionistic fuzzy subgroup of a group $G$.Then $A^{[\alpha,\beta]}$ is a subgroup of a group $G$ , where $M_A(e) \geq \alpha,\ N_A(e) \leq \beta$ and $e$ is the identity element of a group $G$

**Proof:**

Clearly $A^{[\alpha,\beta]} \neq \phi$ as $e \in A^{[\alpha,\beta]}$

Let $x, y \in A^{[\alpha,\beta]}$ be any two elements. Then $M_A(x) \geq \alpha,\ N_A(x) \leq \beta$ and $M_A(y) \geq \alpha,\ N_A(y) \leq \beta$

$\Rightarrow M_A(x) \wedge M_A(y) \geq \alpha$ and $N_A(x) \vee N_A(y) \leq \beta$

As $A$ is Interval valued intuitionistic fuzzy subgroup of $G$.

Therefore $M_A(xy^{-1}) \geq M_A(x) \wedge M_A(y) \geq \alpha$ and $N_A(xy^{-1}) \leq N_A(x) \vee N_A(y) \leq \beta$

$M_A(xy^{-1}) \geq \alpha$ and $N_A(xy^{-1}) \leq \beta$

$\Rightarrow xy^{-1} \in A^{[\alpha,\beta]}$Hence $A^{[\alpha,\beta]}$ is a subgroup of $G$

**CONCLUSION**

As in the theory of Intuitionistic fuzzy set, Level sets are important tool for the development of the subject. Similarly in the theory of Interval valued intuitionistic fuzzy set, $(\alpha,\beta)$-sets are important tools for the development of the subject.

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